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Hard thermal loops with a background plasma velocity

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Abstract

I consider the calculation of the two and three-point functions for QED at finite temperature in the presence of a background plasma velocity. The final expressions are consistent with Lorentz invariance, gauge invariance and current conservation, pointing to a straightforward generalization of the hard thermal loop formalism to this physical situation. I also give the resulting expression for the effective action and identify the various terms.

1 Introduction

The calculation of quantities like the photon polarization tensor, $\Pi_{\mu\nu}$, and the fermion self energy, Σ, for arbitrary values of the photon and fermion momentum ($k_0 = \omega, \vec{k}$), at finite temperature is important for the extraction of many characteristic plasma properties such as the dielectric response, dispersion relations, and calculation of plasma radiation [1].

At finite temperature Lorentz invariance is more involved because of the presence of the background plasma velocity, u^μ. Usually, the calculations are done for the case of a plasma at rest, $u^\mu = (1, 0, 0, 0)$, in which case Lorentz invariance is lost because of the existence of a preferred rest frame. Various quantities can then depend separately on ω and $\vec{k}$.

Applications of finite temperature field theory at realistic situations, however, would need the extension of these results to include an arbitrary (determined by the equations of motion) background plasma velocity. Such would be the case also if one wants to consider the calculation of various properties of the quark gluon plasma state formed in relativistic heavy ion collisions [2].

The inclusion of the plasma velocity effects can be done when one uses a more general thermal distribution function derived from kinetic theory [3, 4]. In the case of fermions, for example, in the place of the ordinary Fermi-Dirac distribution, we have

$$n(p) = \frac{1}{\exp \beta\, |u \cdot p| + 1} \tag{1}$$

where β is the inverse temperature T, and p is the fermion four momentum.

The covariant expressions for the vacuum polarization and the fermion self-energy were given in [4], [6]. Here, besides the two-point functions I will consider the calculation of the vertex correction in QED, in the hard thermal loop approximation, and show that the final expressions satisfy the conditions of gauge invariance that are necessary for the application of the hard thermal loop resummation.

From the resulting expressions I also calculate the general form of the hard thermal loop effective action for QED and give the first order corrections in the nonrelativistic limit, $u^\mu \approx (1, \vec{u})$. In this limit I also give the expression for the dielectric tensor ϵ_{ij}, which will now depend also on u. The corresponding contribution to the effective action will be of the form $\epsilon_{ij}\, E_i\, E_j$, where $\vec{E}$ is the electric field.

There is, also, another term in the resulting effective action, which, as I will show, turns out to be proportional to $\vec{u}\cdot(\vec{E}\,\times\,\vec{B})$, where $\vec{B}$ is the magnetic field. This term is expected to appear in the effective action of electrodynamics in the case of moving dielectrics ([9] and references therein), and was derived there heuristically using symmetry arguments. Although the physical situation here is different, the same term appears, and its coefficient is calculated from the microscopic theory. As shown in [9], the inclusion of this term is important in the consideration of radiative processes, and it should be significant in the applications of the present results as well.

In Sec. 2 I will review and outline the calculation of the two-point functions at finite temperature including the finite plasma velocity effects, in the formalism appropriate to the hard thermal loop resummations. In Sec. 3 I will give the expression for the three-point function and show that, this, as well as the previous expressions, satisfy the constraints imposed by gauge invariance, Lorentz invariance and current conservation. I also discuss the contributions to the effective action and give the interpretation of the resulting effective action in terms of a velocity-dependent dielectric tensor and the additional term mentioned above. Finally, in Sec. 4 I will conclude with some remarks about the possible applications and extensions of these results.

2 Two-point functions with a background plasma velocity

The fermion propagator at finite temperature can be written as

$$S(x,y)=\int\frac{d^4p}{(2\pi)^4}\left[\frac{i(\gamma\cdot p)}{p^2+i\epsilon}-2\pi(\gamma\cdot p)n_p\,\delta(p^2)\right]e^{-ip\cdot(x-y)} \tag{2}$$

where we have neglected the fermion mass in the high temperature limit. We use the generalized distribution function

$$n_p=\frac{1}{e^{\beta|u\cdot p|}+1} \tag{3}$$

and the propagator can be evaluated as

$$\begin{aligned} S(x,y) &= \int\frac{d^3p}{(2\pi)^3}\frac{1}{2E_p}\Big\{\theta(x^0-y^0)\left[\alpha_p(\gamma\cdot p)e^{-ip\cdot(x-y)}+\beta_{p'}(\gamma\cdot p')e^{ip'\cdot(x-y)}\right] \\ &- \theta(y^0-x^0)\left[\beta_p(\gamma\cdot p)e^{-ip\cdot(x-y)}+\alpha_{p'}(\gamma\cdot p')e^{ip'\cdot(x-y)}\right]\Big\} \end{aligned} \tag{4}$$

where

$$\alpha_p = 1 - n_p\,, \quad \beta_p = n_p \tag{5}$$

$$p = (E_p, \vec{p})\,, \;\; p' = (E_p, -\vec{p}) \tag{6}$$

The one-fermion loop contribution to the effective action is

$$\Gamma = \frac{i}{2} \int d^4x\, d^4y\, Tr\left[\gamma \cdot A(x) S(x,y) \gamma \cdot A(y) S(y,x)\right]. \tag{7}$$

We substitute $A_\mu(x) = \int \frac{d^4k}{(2\pi)^4} e^{ik\cdot x} A_\mu(k)$ and after we perform the traces and take $|\vec{k}|$ small compared to $|\vec{q}|$ in the hard thermal loop approximation we get

$$\Gamma = \int_{k,k'} A_\mu(k) A_\nu(k') (2\pi)^4\, \delta^{(4)}(k+k')\, \Pi^{\mu\nu}(k) \tag{8}$$

with the temperature dependent part of the photon polarization tensor

$$\begin{aligned}
&\Pi^{\mu\nu}(k) = -\frac{1}{2} \int \frac{d^3q}{(2\pi)^3} \frac{1}{2p_0 2q_0} \\
&\Big\{\; (n_q - n_p) \frac{8q_0^2 Q^\mu Q^\nu}{p_0 - q_0 - k_0} + (n_{q'} - n_{p'}) \frac{8q_0^2 Q'^\mu Q'^\nu}{p_0 - q_0 + k_0} \\
&-4q_0^2 (Q^\mu Q'^\nu + Q'^\mu Q^\nu - 2g^{\mu\nu}) \left(\frac{n_p + n_{q'}}{p_0 + q_0 - k_0} + \frac{n_{p'} + n_q}{p_0 + q_0 + k_0} \right) \;\Big\}
\end{aligned} \tag{9}$$

where $\vec{p} = \vec{q} + \vec{k}$ and we have the following notations and relations:

$$\begin{aligned}
&Q^\mu = (1, \hat{q}) \quad Q'^\mu = (1, -\hat{q}) \quad \hat{q} = \vec{q}/q_0 \\
&p_0 \approx q_0 + \hat{q} \cdot \vec{k}
\end{aligned} \tag{10}$$

We have also dropped the $i\epsilon$'s been interested here in the real part. Now one can decouple the angular integrals using relations such as

$$\begin{aligned}
&n_p - n_q \approx \frac{dn_q}{dq_0} \left(\frac{u_0\, \hat{q} \cdot \vec{k} - \vec{u} \cdot \vec{k}}{u_0 - \vec{u} \cdot \hat{q}} \right) \\
&\int \frac{d^3q}{(2\pi)^3} \frac{dn_q}{dq_0} f(\hat{q}) = -\int \frac{d^3q}{(2\pi)^3} \frac{2n}{q_0} f(\hat{q}) \\
&\int \frac{d^3q}{(2\pi)^3} \frac{2n_q}{q_0} = \int d\Omega \frac{T^2}{48\pi} \frac{1}{(u_0 - \vec{u} \cdot \hat{q})^2}
\end{aligned} \tag{11}$$

to get the general expressions for the polarization tensor

$$\Pi^{00} = 2m^2 \int \frac{d\Omega}{4\pi} \frac{1}{(u \cdot Q)^3} \left(-u_0 + \frac{u \cdot k}{k \cdot Q} \right) \tag{12}$$

$$\Pi^{0i} = 2m^2 \int \frac{d\Omega}{4\pi} \frac{\hat{q}_i}{(u \cdot Q)^3} \left(-u_0 + \frac{u \cdot k}{k \cdot Q} \right) \tag{13}$$

$$\begin{aligned} \Pi^{ij} = 2m^2 \int \frac{d\Omega}{4\pi} \quad \{ \quad & \frac{\hat{q}_i \hat{q}_j}{(u \cdot Q)^3} \left(-u_0 + \frac{u \cdot k}{k \cdot Q} \right) \\ & -\frac{1}{2} \frac{\hat{q}_i \hat{q}_j + g^{ij}}{(u \cdot Q)^2} \} \end{aligned} \tag{14}$$

where we have defined the photon thermal mass $m^2 = e^2 T^2/6$.

These are the general expressions for the polarization tensor. It is straightforward to verify that they satisfy the condition imposed by current conservation, $k_\mu \Pi^{\mu\nu} = 0$, and use it to get the full expression in a Lorentz covariant form [4]. Here I will give the result in the limit of nonrelativistic u, which will be used in the next section, taking $u^\mu \approx (1, \vec{u})$, and keeping the terms of first order in $\vec{u}$.

$$\begin{aligned} \Pi^{00}/(m^2) &= (-1 + L) + \\ &+ (2k^2 F_1)(\vec{u} \cdot \vec{k}) \end{aligned} \tag{15}$$

$$\begin{aligned} \Pi^{0i}/(m^2) &= (-1 + L)\frac{k_0 k_i}{k^2} + \\ &+ (3k_0 F_2 - 1)\, u_i + \\ &+ (3k_0 F_1 - L + 1)(\vec{u} \cdot \vec{k}) k_i \end{aligned} \tag{16}$$

$$\begin{aligned} \Pi^{ij}/(m^2) &= (k_0 F_1) k_i k_j + (k_0 F_2)\delta_{ij} + \\ &+ (3k_0 G_1 - F_1)(\vec{u} \cdot \vec{k}) k_i k_j + \\ &+ (3k_0 G_2 - F_2)(\vec{u} \cdot \vec{k})\delta_{ij} + \\ &+ (3k_0 G_2)(u_i k_j + u_j k_i) \end{aligned} \tag{17}$$

where

$$L = \frac{1}{2}\frac{k_0}{|\vec{k}|}\ln\left(\frac{k_0 + |\vec{k}|}{k_0 - |\vec{k}|}\right) \tag{18}$$

and

$$F_1 = \frac{1}{2k_0 k_i^4}(3k_0^2 L - k_i^2 L - 3k_0^2) \tag{19}$$

$$F_2 = \frac{1}{2k_0 k_i^2}(k_i^2 L - k_0^2 L + k_0^2) \tag{20}$$

$$G_1 = \frac{1}{6k_i^6}(15k_0^2 L - 9k_i^2 L - 15k_0^2 + 4k_i^2) \tag{21}$$

$$G_2 = \frac{1}{6k_i^4}(3k_i^2 L - 3k_0^2 L + 3k_0^2 - 2k_i^2) \tag{22}$$

The calculation of the fermion self-energy, $\Sigma(k)$, can be done in the same way, using the photon propagator in the Landau gauge,

$$G(x,y) = \int \frac{d^4p}{(2\pi)^4}\left[\frac{i}{p^2 + i\varepsilon} + 2\pi n_p^B\,\delta(p^2)\right] e^{-ip\cdot(x-y)} \tag{23}$$

where,

$$n_p^B = \frac{1}{e^{\beta|u\cdot p|} - 1} \tag{24}$$

The calculations are done similarly in coordinate space using the above two-point function in the form

$$\begin{aligned} G(x,y) &= \int \frac{d^3p}{(2\pi)^3}\frac{1}{2E_p}\Big\{\theta(x^0 - y^0)\left[(1 + n_p^B)e^{-ip\cdot(x-y)} + n_{p'}^B e^{ip'\cdot(x-y)}\right] \\ &- \theta(y^0 - x^0)\left[n_p^B e^{-ip\cdot(x-y)} + (1 + n_{p'}^B)e^{ip'\cdot(x-y)}\right]\Big\} \end{aligned} \tag{25}$$

and the final result for the fermion self-energy is

$$\Sigma(k) = m_f^2 \int \frac{d\Omega}{4\pi}\frac{1}{(u\cdot Q)^2}\frac{\gamma\cdot Q}{k\cdot Q} \tag{26}$$

with the fermion thermal mass $m_f^2 = e^2\,T^2/8$.

This can be evaluated as

$$\Sigma(k) = (\Sigma_1\,k^\mu + \Sigma_2\,u^\mu)\,\gamma_\mu \tag{27}$$

where

$$\Sigma_1 = -\frac{m_f^2}{K^2}\,(1-\tilde{L}) \qquad \Sigma_2 = \frac{m_f^2}{K^2}\,(\Omega - \frac{k^2}{\Omega}\tilde{L}) \tag{28}$$

with

$$\Omega = (u\cdot k), \quad K = \sqrt{(u\cdot k)^2 - k^2}, \quad \tilde{L} = \frac{\Omega}{2K}\,\ln\left(\frac{\Omega+K}{\Omega-K}\right) \tag{29}$$

The poles of the effective fermion propagator

$$S(k) = \frac{1}{\gamma\cdot k - \Sigma(k)} \tag{30}$$

give two branches for the dispersion relations in terms of Ω and K, which are the appropriate relativistic generalizations of ω and $|\vec{k}|$. These two branches $\Omega_\pm(K)$ are the solutions of

$$(1-\Sigma_1)\Omega - \Sigma_2 = \pm(1-\Sigma_1)K \tag{31}$$

and give the usual dispersion relations [5, 6, 7]. In particular $\Omega_-(K)$ has a minimum for non-zero K and describes the usual plasmino mode.

3 The three-point funtion and the effective action

The vertex correction of the three point function, $\Gamma^\mu(k_1,k_2)$, with two external fermion momenta k_1 and k_2, can be extracted from

$$\int d^4x\, d^4y\, d^4z\, \psi(x)S(x,y)A^\mu(y)S(y,z)G(z,x)\bar{\psi}(z) \tag{32}$$

using the expressions (4) and (25) and making the same hard thermal loop approximations we get

$$e\,\Gamma^\mu(k_1,k_2) = m_f^2 \int \frac{d\Omega}{4\pi}\,\frac{1}{(u\cdot Q)^2}\frac{Q^\mu\,(\gamma\cdot Q)}{(k_1\cdot Q)\,(k_2\cdot Q)} \tag{33}$$

This can also be written covariantly as:

$$e\,\Gamma^\mu(k_1,k_2) = \alpha_1\,k_1^\mu + \alpha_2\,k_2^\mu + \alpha_3\,u^\mu. \tag{34}$$

where the expressions α_1, α_2 and α_3 are covariant, scalar functions of k_1, k_2, u, and the gamma matrices, that satisfy

$$\begin{aligned}
&\alpha_1\, k_1^2 + \alpha_2\, (k_1 \cdot k_2) + \alpha_3\, (u \cdot k_1) = \Sigma(k_2) \\
&\alpha_1\, (k_1 \cdot k_2) + \alpha_2\, k_2^2 + \alpha_3\, (u \cdot k_2) = \Sigma(k_1) \\
&\alpha_1\, (u \cdot k_1) + \alpha_2 (u \cdot k_2) + \alpha_3 = J(k_1, k_2)
\end{aligned} \tag{35}$$

with $\Sigma(k)$ given by (27) and

$$J(k_1, k_2) = \int d\Omega\, \frac{(\gamma \cdot Q)}{(u \cdot Q)(k_1 \cdot Q)(k_2 \cdot Q)} \tag{36}$$

It is now straightforward to see from these expressions, or directly from (26) and (33) that the Ward identity

$$e(k_1 + k_2)_\mu\, \Gamma^\mu(k_1, k_2) = \Sigma(k_1) + \Sigma(k_2) \tag{37}$$

that is essential for maintaining gauge invariance in the hard thermal loop resummation, is satisfied.

Now I will describe the resulting hard thermal loop effective action. First, as far as the fermion terms are concerned, it is easy to see from (26) that

$$\mathcal{L}_f = m_f^2\, \bar{\psi}(x) \int \frac{d\Omega}{4\pi} \frac{1}{(u \cdot Q)^2} \frac{\gamma \cdot Q}{(Q \cdot D)}\, \psi(x) \tag{38}$$

is the appropriate gauge invariant expression for the effective fermion Lagrangian in coordinate space, with $D_\mu = \partial_\mu + eA_\mu$ the covariant derivative, which, upon expanding the denominator, gives the effective three-point vertex (33) as the first correction.

Now, one can apply the usual gauge invariance arguments and power counting rules, to claim that higher order terms in the expansion of (38) will give the relevant contributions of the N-point functions in the hard thermal loop approximation [8].

As far as the photon effective action is concerned, one can make a similar guess from (12), (13), (14), and obtain an expression that in the rest frame $u^\mu = (1, \vec{0})$ reduces to the known result [8]. Here I will try to give a physical interpretation in the nonrelativistic limit. In order to do that first we write the gauge potential as [10]

$$A^\mu(k) = k^\mu \theta + \phi^\mu + \beta^\mu \tag{39}$$

where θ is a term that can be gauged away to zero,

$$\phi^\mu = \left(-\frac{k_i^2}{k^2}\Phi, -\frac{k_0 k_i}{k^2}\Phi\right) \tag{40}$$

is the electrostatic part and

$$\beta^\mu = (0, \beta_i) \quad k_i \beta_i = 0 \tag{41}$$

is the transverse part. In terms of these and using the relations $k_\mu \Pi^{\mu\nu} = 0$ and $k_i \beta_i = 0$ we get from (8)

$$\begin{aligned}\Gamma = \quad & m^2 \int_k \Phi(k)\Phi(-k)\left[(-1+L) + (2k_i^2\, F_1)(\vec{u}\cdot\vec{k})\right] + \\ + \quad & \Phi(k)\beta_i(-k)\left[(1-3k_0 F_2)u_i\right] + \\ + \quad & \beta_i(k)\beta_j(-k)\left[(k_0\, F_2)\delta_{ij} + (3k_0 G_2 - F_2)(\vec{u}\cdot\vec{k})\delta_{ij}\right]\end{aligned} \tag{42}$$

This expression can be written alternatively as

$$\Gamma = \int_k \frac{1}{2}\,\epsilon_{ij}\, E_i(k) E_j(-k) \;+ \lambda\, \vec{u}\cdot\left(\vec{E}(k)\times\vec{B}(-k)\right) \tag{43}$$

where the general form of the dielectric tensor that is consistent with Maxwell's equations is

$$\begin{aligned}\epsilon_{ij} \;\; = \;\; & \left[\epsilon_1 + \epsilon_3(\vec{u}\cdot\vec{k})\right]\frac{k_i k_j}{k_i^2} + \\ + \;\; & \left[\epsilon_2 + \epsilon_4(\vec{u}\cdot\vec{k})\right]\left(\delta_{ij} - \frac{k_i k_j}{k_i^2}\right)\end{aligned} \tag{44}$$

and the coefficients ϵ_{1-4} and λ are calculated from (42) to be

$$\epsilon_1 = \frac{2m^2}{k_i^2}(1-L) \tag{45}$$

$$\epsilon_2 = -\frac{2m^2}{k_0}\, F_2 \tag{46}$$

$$\epsilon_3 = -4m^2\, F_1 \tag{47}$$

$$\epsilon_4 = \frac{2m^2}{k_i^2\, k_0^2}\left[2k_0 - 3k_0\, k_i^2\, G_1 - F_2\,(6k_0^2 - k_i^2)\right] \tag{48}$$

$$\lambda = \frac{2m^2}{k_i^2}\,(1 - 3k_0\,F_2) \tag{49}$$

The terms ϵ_1 and ϵ_2 agree with the known values of the longitudinal and transverse dielectric tensor in the heat bath rest frame [1], and ϵ_3 and ϵ_4 give the first order correction in $\vec{u}$.

As I mentioned in the introduction the additional term, proportional to λ, is expected to be there in the case of electrodynamics of moving dielectrics ([9] and references therein). It was derived there using symmetry arguments, and its value was related to that of the dielectric function. Here, of course, the physical situation is different, however, a similar term appears and its value is calculated from the microscopic theory. In any case this term would be important in the calculation of physical properties and radiative processes.

4 Comments

In this work I described some results on the dependence of the high temperature effective action on the background plasma velocity. These and similar results, combined with some phenomenological model regarding the plasma evolution, would be presumably relevant in the case of calculations of physical quantities at finite temperature. I showed that, in the case of QED, the techniques of hard thermal loop effective actions can be applied. Similar calculations, with appropriate hard thermal loop resummations, can be done for the effective action in finite temperature QCD, with possible applications in the theory of quark gluon plasma.

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